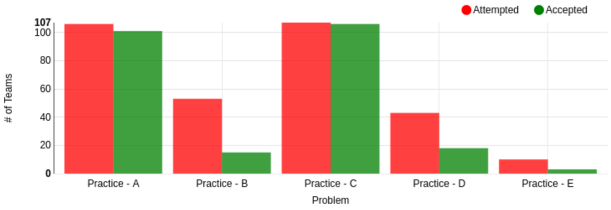
Problem Analysis Session

SWERC judges

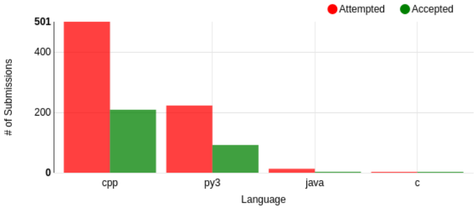
27/01/2024

Statistics

Number of submissions: about 620

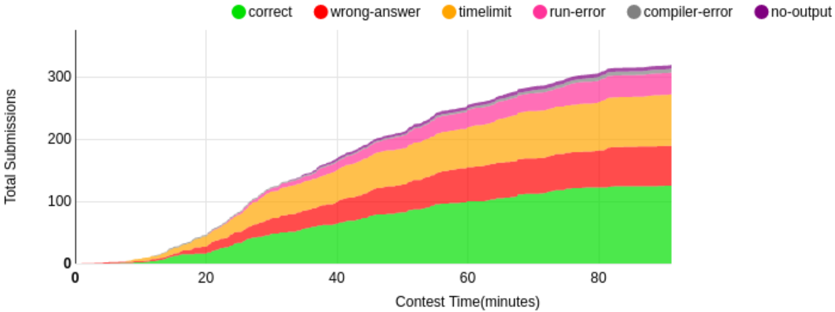


Number of clarification requests: 98 (0 answered "No comment.")



A: Ascending hike

Solved by 101 teams before freeze.
First solved after 7 min by **INSA Rouen**.



A: Ascending hike

Problem

Compute the largest span of a contiguous increasing sub-sequence.

Solution – Linear time & constant space

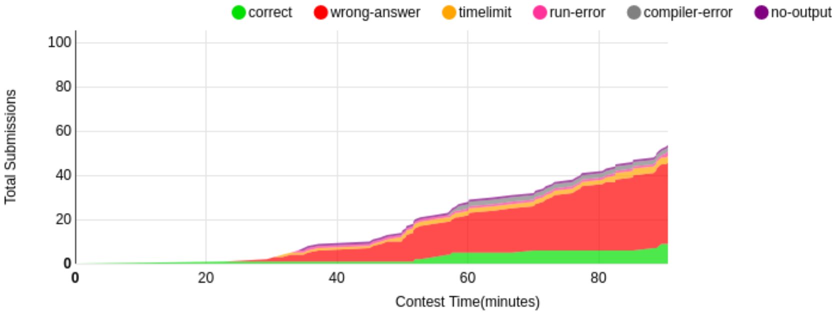
Compute the current elevation gain

$$G_k = \max\{H_k - H_i : H_i < H_{i+1} < \ldots < H_k\}$$
$$= \begin{cases} 0 & \text{if } k = 1 \text{ or } H_k < H_{k-1}, \\ G_{k-1} + (H_k - H_{k-1}) & \text{otherwise,} \end{cases}$$

and output the largest G_k.

B: Close scores

Solved by 16 teams before freeze.
First solved after 22 min by **ETH Zurich**.



B: Close scores

Problem

Given a multigraph G, orient it so as to minimize

$$\max_{v \in \text{ vertices of } G} |d^+(v) - d^-(v)|$$

Solution – Linear time

Suppose G is connected.

If G has a solution with score 0, every vertex has even degree. Thus, G has an Eulerian cycle.

If G has an Eulerian cycle $u_1 \to u_2 \cdots \to u_N \to u_1$, orient the edge $u_i u_{i+1}$ as $u_i \to u_{i+1}$. This achieves score 0.

B: Close scores

Problem

Given a multigraph G, orient it so as to minimize

$$\max_{v \in \text{ vertices of } G} |d^+(v) - d^-(v)|$$

Solution – Linear time

Suppose G is connected.

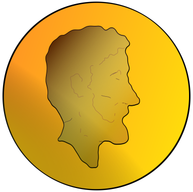
G has a solution with score 0 if and only if it has an Eulerian cycle.

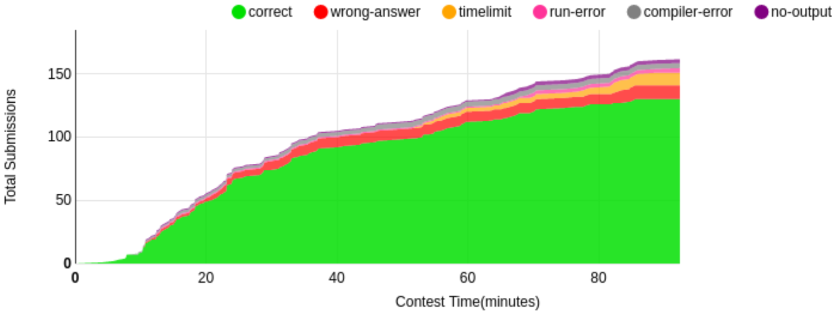
Otherwise, we can achieve score 1 as follows:

- Add a vertex connected to vertices of odd degree,
- The obtained multigraph has only even degrees, thus has an Eulerian cycle, thus can achieve score 0,
- Removing the added vertex deteriorates the score by 1.

C: Everyone is a winner

Solved by 106 teams before freeze.
First solved after 4 min by **Telecom Paris**.





C: Everyone is a winner

Problem

Is it possible to give out M medals to K kids such that they each get the same number of medals?

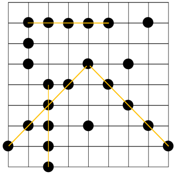
Solution – Constant time & space

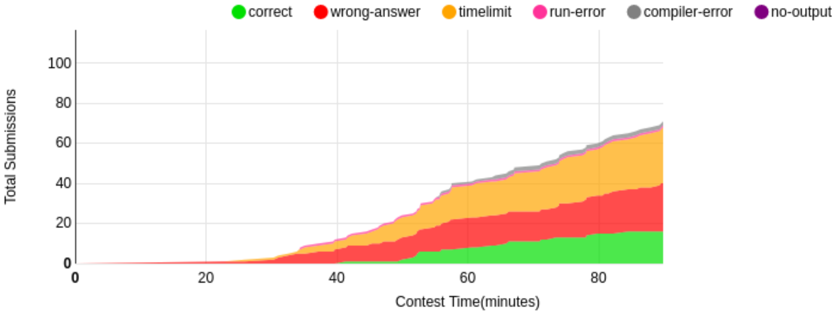
Output 'Yes' in case

$$M \equiv 0 \pmod{K}.$$

D: Five in a row

Solved by 20 teams before freeze.
First solved after 41 min by **Universta di Pisa**.





D: Five in a row

Problem

Find the first five K-dimensional stones that are in a row.

Solution – $O(N^2K)$ time & $O(NK)$ space

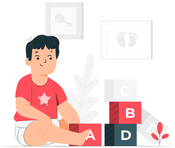
As $N \leqslant 2000$, we cannot use a $O(N^5)$ algorithm to confirm if there are five in a row.

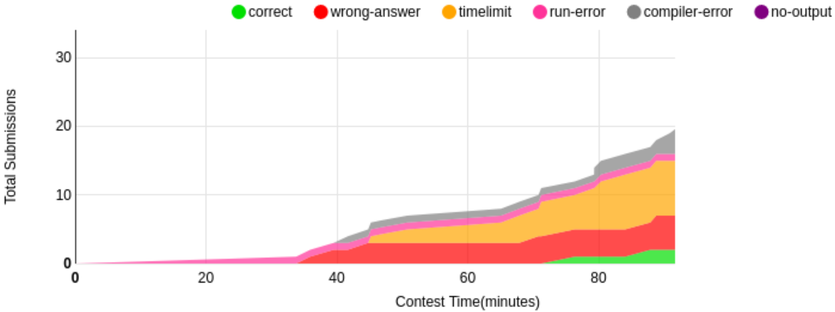
Therefore, we need to use some customized hash table to store all the stones. When reading a new stone, we enumerate all the existing stones as its immediate neighbour in the unbroken line, and see if it can be extended to five stones.

Alternatively, we can use Trie tree to store the stones. Then checking if a stone exists will also take $O(K)$ time. In both cases, the time complexity is $O(N^2K)$.

E: Wooden blocks

Solved by 3 teams before freeze.
First solved after 76 min by **ETH Zurich**.



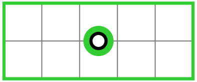


E: Wooden blocks

Problem

Count configurations obtained by stacking in a stable manner a sequence of N blocks.

Construction strategy:



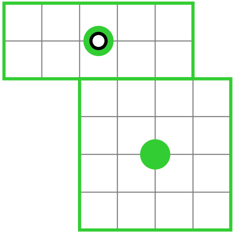
$\text{Area}_3 = 10 \quad 2x_3 = 5 \quad 2x_{\geqslant 3} = 5$

E: Wooden blocks

Problem

Count configurations obtained by stacking in a stable manner a sequence of N blocks.

Construction strategy:



$\text{Area}_3 = 10 \quad 2x_3 = 5 \quad 2x_{\geqslant 3} = 5$

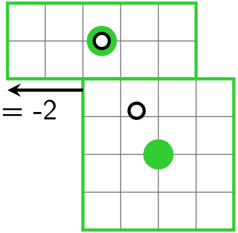
$\text{Area}_2 = 16 \quad 2x_2 = 4$

E: Wooden blocks

Problem

Count configurations obtained by stacking in a stable manner a sequence of N blocks.

Construction strategy:



$\delta_2 =$ -2

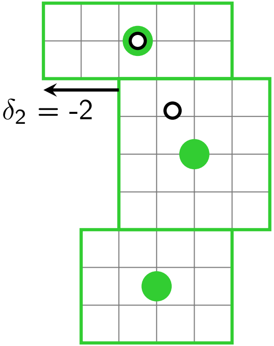
$\text{Area}_3 = 10 \quad 2x_3 = 5 \quad 2x_{\geqslant 3} = 5$

$$\text{Area}_2 = 16 \quad 2x_2 = 4 \quad 2x_{\geqslant 2} = \frac{10 \times 2(x_3 + \delta_2) + 16 \times 2x_2}{10 + 16}$$

Problem

Count configurations obtained by stacking in a stable manner a sequence of N blocks.

Construction strategy:



$\text{Area}_3 = 10 \quad 2x_3 = 5 \quad 2x_{\geqslant 3} = 5$

$\text{Area}_2 = 16 \quad 2x_2 = 4 \quad 2x_{\geqslant 2} = \dfrac{10 \times 2(x_3 + \delta_2) + 16 \times 2x_2}{10 + 16}$

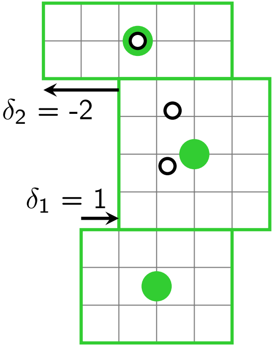
$\text{Area}_1 = 12 \quad 2x_1 = 4$

E: Wooden blocks

Problem

Count configurations obtained by stacking in a stable manner a sequence of N blocks.

Construction strategy:



$\text{Area}_3 = 10 \quad 2x_3 = 5 \quad 2x_{\geqslant 3} = 5$

$\text{Area}_2 = 16 \quad 2x_2 = 4 \quad 2x_{\geqslant 2} = \dfrac{10 \times 2(x_3 + \delta_2) + 16 \times 2x_2}{10 + 16}$

$\text{Area}_1 = 12 \quad 2x_1 = 4 \quad 2x_{\geqslant 1} = \dfrac{26 \times 2(x_{\geqslant 2} + \delta_1) + 12 \times 2x_1}{26 + 12}$

Induction formula for numerator $X_i / \text{Area}_{\geqslant i} = 2\, x_{\geqslant i}$:

$$X_i = X_{i+1} + \delta_i \text{Area}_{\geqslant i+1} + 2x_i \text{Area}_i = X_{i+1} + \delta_i \text{Area}_{\geqslant i+1} + w_i \text{Area}_i$$

E: Wooden blocks

Induction formula for numerator $X_i / \text{Area}_{\geqslant i} = 2\,x_{\geqslant i}$:

$$X_i = X_{i+1} + \delta_i \text{Area}_{\geqslant i+1} + 2x_i \text{Area}_i = X_{i+1} + \delta_i \text{Area}_{\geqslant i+1} + w_i \text{Area}_i$$

Implementation – Time $\mathcal{O}(N^2\,W^3 H)$ & space $\mathcal{O}(N\,W^2 H)$

Maintain the list of possible X_is, with multiplicities:

1. Avoid floating-point numbers
   - may lead to WA because of rounding errors
2. Avoid hashmaps:
   - slows things down, may lead to TLE
3. Just use a good old array!