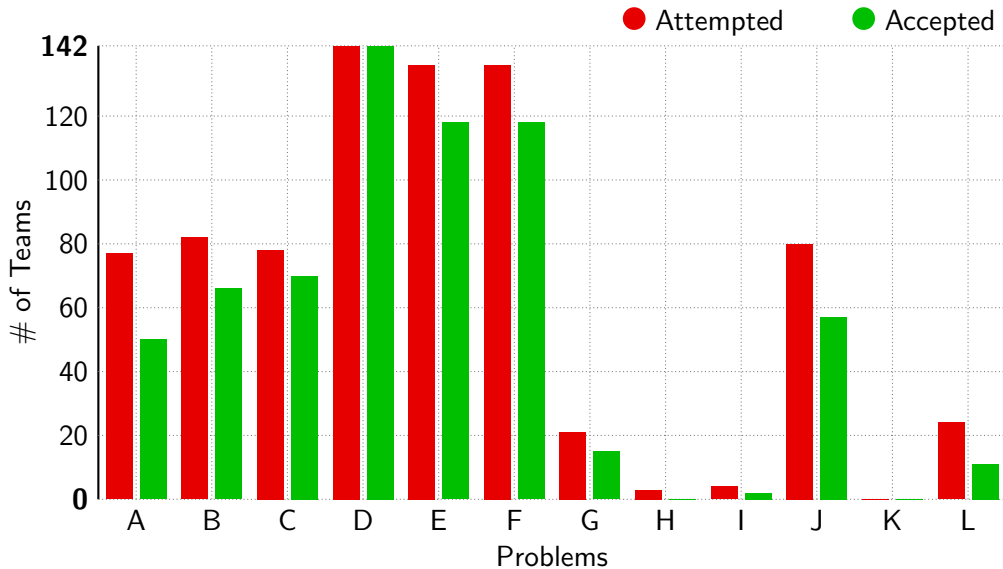
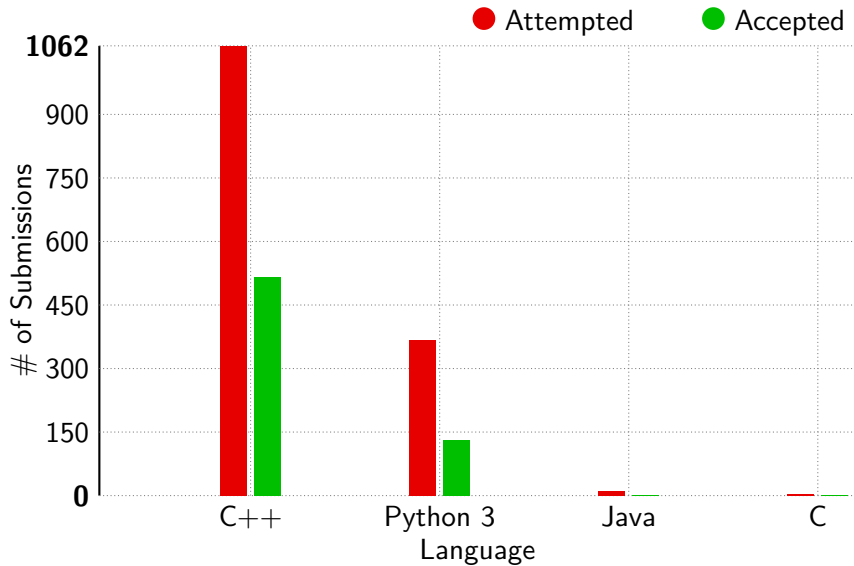


SWERC 2025

Problem Analysis

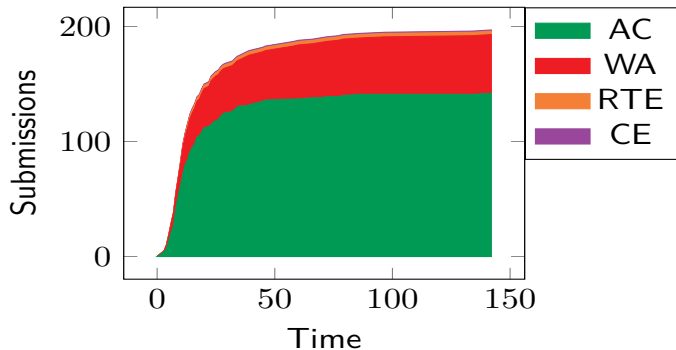
November 23, 2025





D: Dungeon Equilibrium

Solved by 142 teams before freeze.
First solved after 3 min by UXT.



D: Dungeon Equilibrium

Statement (summary).

We are given an array $a_1, \dots, a_n$ ($0 \leq a_i \leq n$). A level is *balanced* if, for every monster type x that appears, the total number of occurrences of x in the array is exactly x . We may delete elements. Compute the minimum number of deletions needed to make the level balanced.

D: Dungeon Equilibrium

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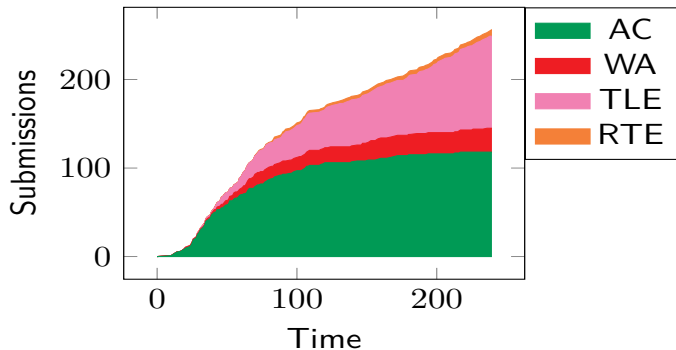
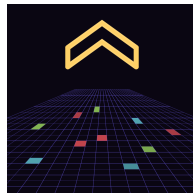
Algorithm.

- 1 Count frequencies $f[x]$ for all $x \in [0, n]$.
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Complexity. $O(n)$ time (frequency array of size $n+1$). Slower solutions (e.g., $O(n^2)$) are allowed.

E: Expansion Plan 2

Solved by 118 teams before freeze.
First solved after 10 min by TempName.



Statement (summary).

Initially, only cell $(0, 0)$ is black. With a character 4, the 4 cells touching a black cell orthogonally become black. With a character 8, the 8 cells touching a black cell orthogonally or diagonally become black. Given a string s , answer queries (l, r, x, y) : is cell (x, y) black after the commands in substring $[l, r]$?

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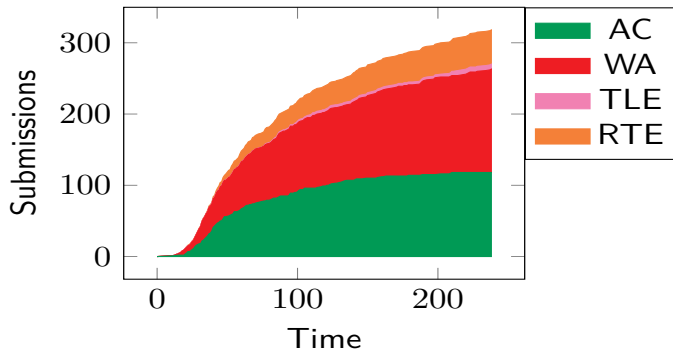
Complexity. $O(n + q)$ time.

F: Factory Table

Solved by 118 teams before freeze.

First solved after 11 min by Segfault go BRRRR.

×	1	2	3	4
1	1	2	3	4
2	2	4	6	8
3	3	6	9	12
4	4	8	12	16



Statement (summary).

The k -th unrolled table is $[1 \cdot 1, 1 \cdot 2, \dots, 1 \cdot k, 2 \cdot 1, 2 \cdot 2, \dots, 2 \cdot k, \dots, k \cdot 1, k \cdot 2, \dots, k \cdot k]$. We are given a subarray of an unrolled table, with length ≥ 2 . Find the minimum possible k .

Key observations.

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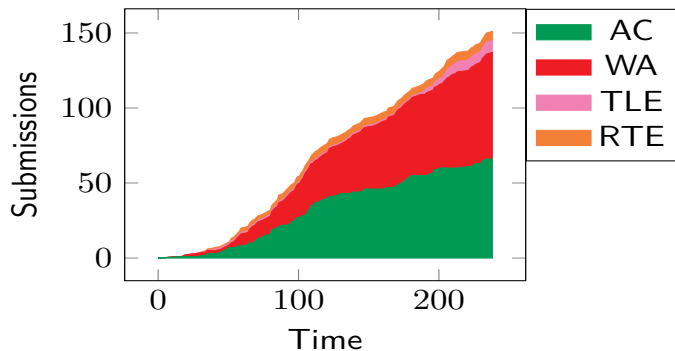
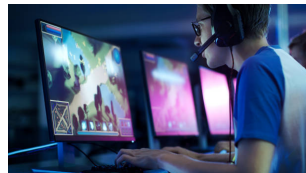
Algorithm.

- ① For each element, find its row $r[i]$ and its column $c[i]$.
- ② Among all the $r[i]$ and $c[i]$, output the largest.

Complexity. $O(n)$ time per test case.

B: Billion Players Game

Solved by 66 teams before freeze.
First solved after 17 min by UniBois.



B: Billion Players Game

Statement (summary).

There is a hidden integer x in the interval $[l, r]$. We are given some offers $a_1, a_2, \dots, a_n$. For each of them, we can choose to either get $x - a_i$ coins, $a_i - x$ coins, or nothing. Maximize the number of coins in the worst case.

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Optimal strategy. Let's color a_i red if we get $x - a_i$ coins.

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There exists an optimal strategy where a prefix of elements (after sorting) is red, and a suffix of elements is blue.

Sketch of proof. Use exchange argument. For example, if $y \leq z$, y is white and z is red, then we get $x - z$ coins. If we make y red and z white, we make $x - y$ coins instead.

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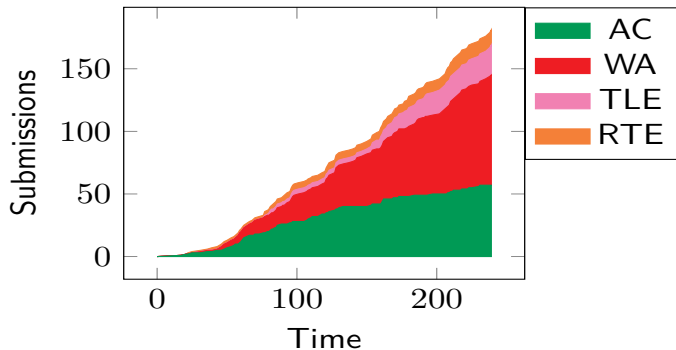
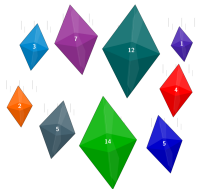
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Complexity. $O(n)$ time per test case ($O(n \log n)$ is also possible, for example using ternary search).

J: Jewels Building

Solved by 57 teams before freeze.
First solved after 17 min by TempName.



Statement (summary).

We are given two arrays $a_1, a_2, \dots, a_n$ and $b_1, b_2, \dots, b_m$. Determine whether it is possible to turn a into b in a finite number of operations.

Operation: pick a subarray of length $x \geq 1$ containing equal elements, and replace it with x .

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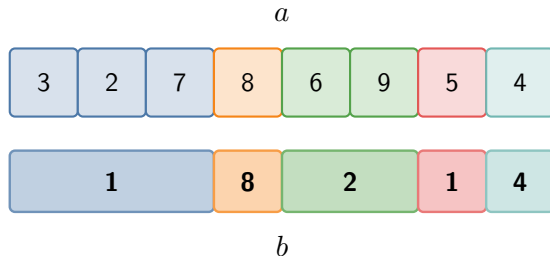
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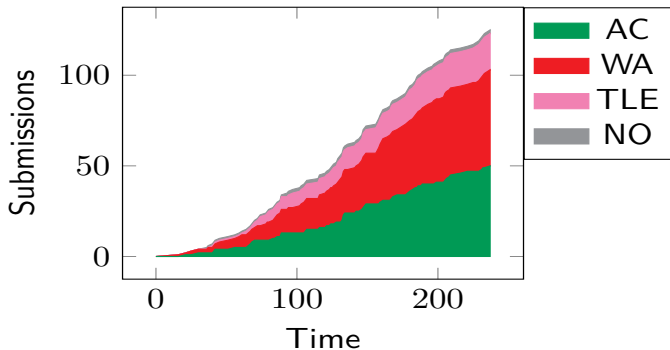
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Complexity. $O(nm)$ time per test case.

A: Adjusting Drones

Solved by 50 teams before freeze.
First solved after 21 min by ENS de Lyon 3.



A: Adjusting Drones

Statement (summary).

Given an array, on each operation we increase simultaneously by 1 all the elements which are equal to some previous element. After how many operations every element appears at most k times?

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Assume the process stops when all the elements are distinct. Let's find the configuration $b_1, b_2, \dots, b_n$ at that point. If some elements are equal, the **rightmost** does not increase.

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A: Adjusting Drones

t	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
0	3	3	3	4	5	5	6	6	6	6	6	7	7	7	13	13
1	4	4	3	4	6	5	7	7	7	7	6	8	8	7	14	13
2	5	5	3	4	7	5	8	8	8	8	6	9	8	7	14	13
3	6	6	3	4	8	5	9	9	9	9	6	9	8	7	14	13
4	7	7	3	4	9	5	10	10	10	10	6	9	8	7	14	13
5	8	8	3	4	10	5	11	11	11	10	6	9	8	7	14	13
6	9	9	3	4	11	5	12	12	11	10	6	9	8	7	14	13
7	10	10	3	4	12	5	13	12	11	10	6	9	8	7	14	13
8	11	11	3	4	13	5	14	12	11	10	6	9	8	7	14	13
9	12	12	3	4	14	5	15	12	11	10	6	9	8	7	14	13
10	13	13	3	4	15	5	15	12	11	10	6	9	8	7	14	13
11	14	14	3	4	16	5	15	12	11	10	6	9	8	7	14	13
12	15	15	3	4	16	5	15	12	11	10	6	9	8	7	14	13
13	16	16	3	4	16	5	15	12	11	10	6	9	8	7	14	13
14	17	17	3	4	16	5	15	12	11	10	6	9	8	7	14	13
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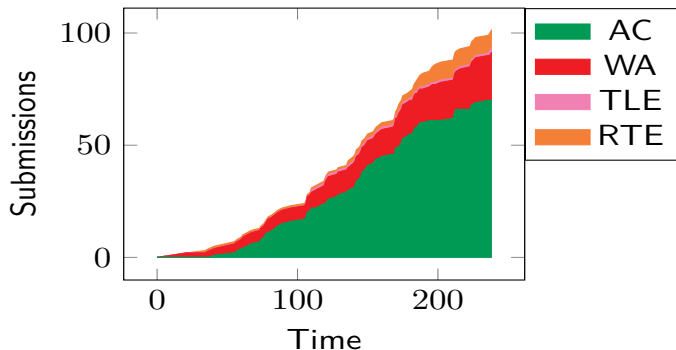
Complexity. $O(n \log n)$ time (there also exist “magical” $O(n)$ solutions!)

C: Chamber of Secrets 2

Solved by 70 teams before freeze.

First solved after 41 min by Ctrl+Alt+DeIETHe.

LEVEL 
UNLOCKED!



C: Chamber of Secrets 2

Statement (summary).

We have n arrays of length m built as follows:

- 1 start from a secret key $[p_1, p_2, \dots, p_{nm/2}]$ (a permutation);
- 2 concatenate p with itself;
- 3 split the resulting array into n consecutive blocks, i.e., disjoint subarrays of length m ;
- 4 shuffle these arrays.

Find a possible secret key $[p_1, p_2, \dots, p_{nm/2}]$.

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C: Chamber of Secrets 2

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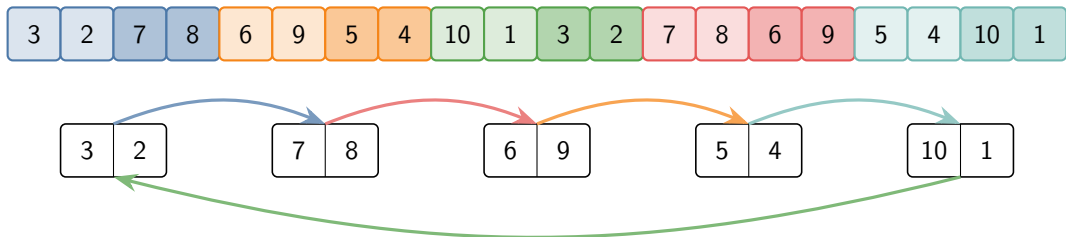
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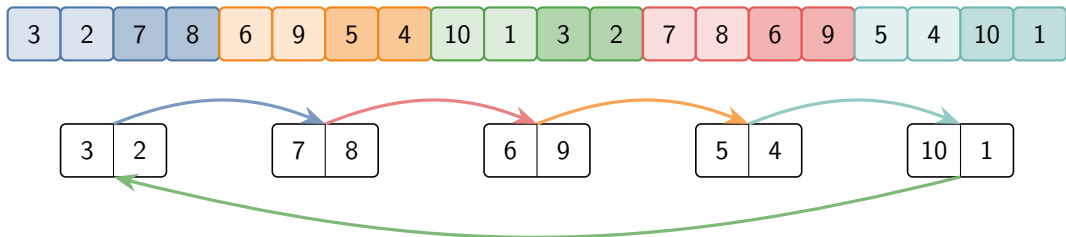
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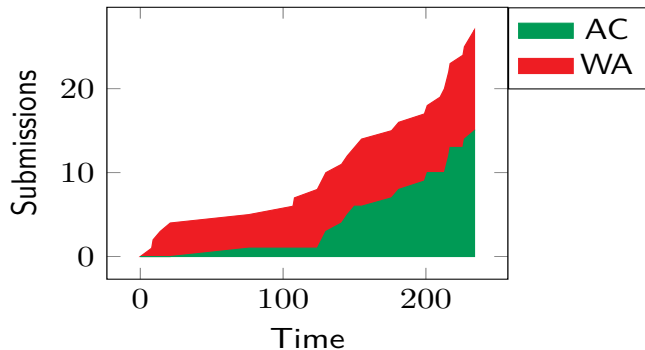
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Complexity. $O(nm)$ time per test case.

Solved by 15 teams before freeze.
First solved after 76 min by Team 2.



Statement (summary).

We want to make our skill $\geq n$ using a limited amount of robocoins. We can perform operations (y, l) with cost l and additional cost 1000 if y is larger than the previous one. The operation makes our skill $y + l$ if it was exactly y .

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Use operations $1, 2, \dots, n - 1$. Unfortunately, they are too “increasing”.

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Naive strategy 1.

Use operations $1, 2, \dots, n - 1$. Unfortunately, they are too “increasing”.

Naive strategy 2.

Use operations $n - 1, n - 2, \dots, 1$. Unfortunately, they are too long.

Better strategy.

- Fix our skill modulo 2 using operations $n - 1, n - 3, n - 5, \dots$

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The cost is $O(n \log n)$ ($\approx \log_2 n$ layers, $O(n)$ per layer). We use very few “increasing” operations (1 per layer): we can try to use more (and maybe use less layers).

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Generalization.

- Fix our skill modulo 3 using operations $n - 2, n - 5, n - 8, \dots, n - 1, n - 4, n - 7, \dots$

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- Fix our skill modulo 3 using operations $n - 2, n - 5, n - 8, \dots, n - 1, n - 4, n - 7, \dots$
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Now we have $\approx \log_3 n$ layers and 2 “increasing” operations per layer.

G: Git Gud

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24		26	27
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21		23	24		26	27
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	3		6		9		12		15		18		21		24		27
	3		6		9		12		15		18				24		27
	3		6		9				15		18				24		27
			6		9				15		18				24		27
			6		9				15		18						27
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			6		9						18						27
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[illegible]

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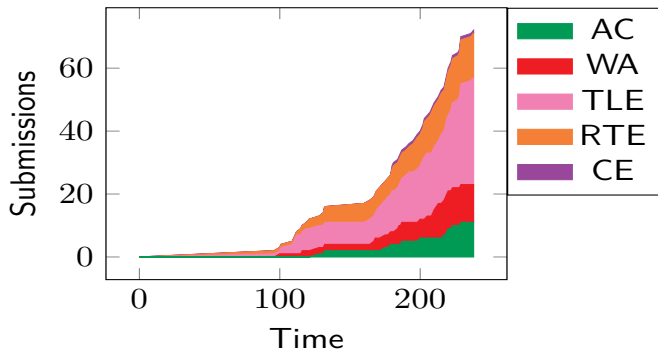
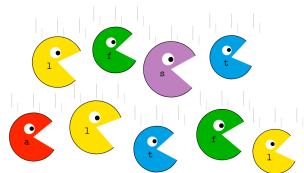
if $n = 250\,000$.

If n is smaller, we can pretend it is 250 000.

L: LFS

Solved by 11 teams before freeze.

First solved after 128 min by Northern Spy.



Statement (summary).

Given a string s , answer q queries: what is the maximum length of a substring appearing with maximum frequency in the substring $[l, r]$?

Key observations. Let $\alpha = 26$ (size of the alphabet).

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- The maximum frequency is reached by a substring which is 1 character long.
- A substring which ends with a character c can be extended to the right if and only if $s[r] \neq c$, and every occurrence of c in $[l, r]$ is followed by the same character d .
- Make an edge for every such pair (c, d) of characters. There are $O(\alpha)$ such pairs (for each c , there exists at most one valid d).

Algorithm.

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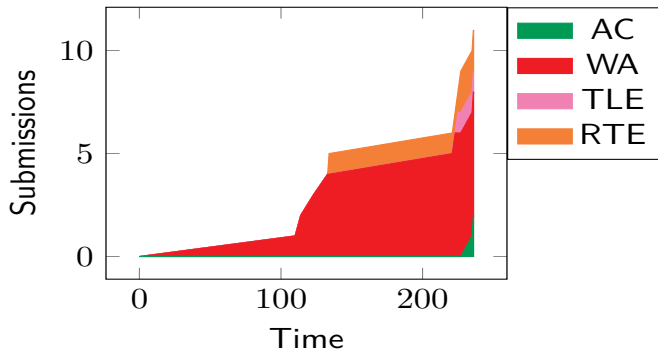
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Complexity. $O((n + q)\alpha)$ time (additional $O(\log n)$ factors with low constant are allowed).

I: Isaac's Queries

Solved by 2 teams before freeze.

First solved after 235 min by Segfault go BRRRR.



I: Isaac's Queries

Statement (summary).

There is a hidden array, generated randomly. For each subarray $[l, r]$, we want to find the most significant bit of $a_l \oplus a_{l+1} \oplus \dots \oplus a_r$. We can ask the answer for some $[l, r]$, with cost $\frac{1}{r-l+1}$. Find all the answers with limited total cost.

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If $g(l, m) \neq g(m + 1, r)$, then $g(l, r) = \max(g(l, m), g(m + 1, r))$. Otherwise, we know that $g(l, r) < g(l, m)$.

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If $y > l$ and $g(l, y) \neq g(r + 1, y)$, then $g(l, r) = \max(g(l, y), g(r + 1, y))$.

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In order to ask $g(l, r)$, it must be less than $g(1, r), g(2, r), \dots, g(l-1, r)$. So the probability of asking it is $O(1/l)$. So the average cost is

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In practice, the average cost is ≈ 20 .

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Let's build a trie containing the a_i . Specifically, each node corresponds to a prefix of the binary representation, and contains positions i such that a_i has that prefix. It is not possible to recover the a_i uniquely, but we can set bits to 0 without loss of generality if they do not impact any queries.

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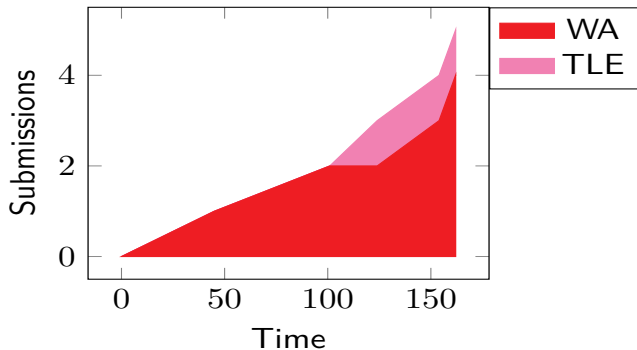
Specifically, suppose that a node contains positions $p_1, p_2, \dots, p_k$. In order to recurse, we need to ask queries between some pairs. We choose this pairs greedily, by calculating a minimum spanning tree (the edges have cost $\frac{1}{p_j - p_i + 1}$).

Cost of Solution 2.

The same analysis of Solution 1 gives an upper bound. However, this solution is much more efficient (average cost ≈ 9).

H: Hyper Smawk Bros

Not solved before freeze.



Statement (summary).

You and Bob want to make an integer $n \leq 0$. You alternately subtract an integer in $[1, m]$ from n , but it must be different from the one just chosen by the opponent. Given n, m , determine whether you can force a win. Answer multiple test cases.

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Losing positions.

Let (n, l) ($0 \leq m$) be losing if you lose if it's your turn, the current integer is n , and you cannot subtract l (if l is 0, you can subtract everything).

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Bound on the evil n (I).

Claim: there are $O(N/m)$ evil n . Specifically, let $v_1, v_2, \dots, v_k$ be the evil $n \leq N$. Then, $v_i - v_{i-1} \geq m + 1$ (otherwise you would be able to move from an evil n to another, contradiction).

Structure of losing positions and solution in $O(t(N + m))$.

If (n, l) is losing, then $(n + l, l')$ with $l' \neq l$ is winning. This means that the number of losing l for a fixed n is either 0, 1, or $m + 1$. Let's call n “good”, “neutral”, and “evil”, respectively.

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Even m .

For even m , only the multiples of $m + 1$ are evil. In order to win, you can ensure that the last two moves have sum $m + 1$. Since $m + 1$ is odd, the last move will never be the same as the second last.

H: Hyper Smawk Bros

Structure of losing positions and solution in $O(t(N + m))$.

If (n, l) is losing, then $(n + l, l')$ with $l' \neq l$ is winning. This means that the number of losing l for a fixed n is either 0, 1, or $m + 1$. Let's call n “good”, “neutral”, and “evil”, respectively. Note that the losing states are $O(N + m)$ in total.

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From now, let's only consider odd m .

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Bound on the evil n (II).

Let $v_1, v_2, \dots, v_k$ be the evil $n \leq N$. Then, $m + 1 \leq v_i - v_{i-1} \leq m + 2$.

H: Hyper Smawk Bros

n	losing last moves l (if $m = 9$)
21	[0, 1, 2, 3, 4, 5, 6, 7, 8, 9]
22	[1]
23	[]
24	[]
25	[]
26	[5]
27	[6]
28	[]
29	[8]
30	[9]
31	[5]
32	[0, 1, 2, 3, 4, 5, 6, 7, 8, 9]
33	[]
34	[2]
35	[3]

Our goal.

Let $v_1, v_2, \dots, v_k$ be the evil $n \leq N$. We want to find them in $O(k) = O(N/m)$. In this way, we can solve the problem for all m in $O(\sum_{1 \leq m \leq M} N/m) = O(N \log M)$.

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Finding v_{i+1} .

We have $v_1, v_2, \dots, v_i$, and we want to find v_{i+1} in $O(1)$. It is enough to check whether $v_i + m + 1$ is evil. It's easier to check whether it is **not** evil, i.e., whether there exists a winning move.

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So the function calls itself $\lesssim 7/8$ times on average. This is fast enough for all m (it can be tested locally).

K: Keygen 3

Not solved before freeze.



Statement (summary).

Print all the bitonic permutations of length n with m cycles. If there are more than 2000 of them, print 2000 of them.

Minor modification.

For convenience, let's find “anti-bitonic” permutations instead (decreasing, then increasing).

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```
vector<int> f(vector<int> p) {  
    int n = p.size();  
    reverse(p.begin(), p.end());  
    for (auto &u : p) u = n - u + 1;  
    return p;  
}
```

K: Keygen 3

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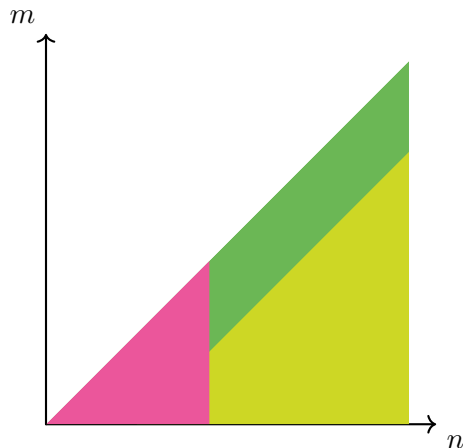
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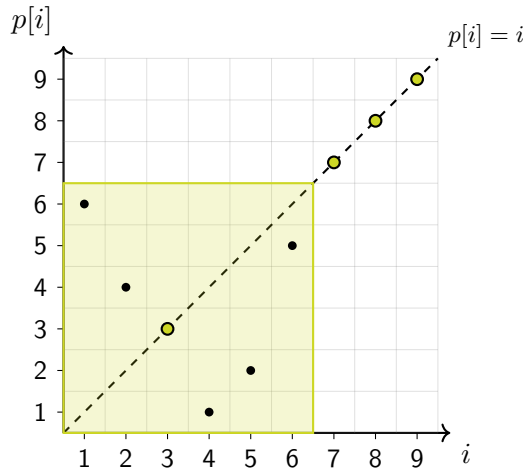
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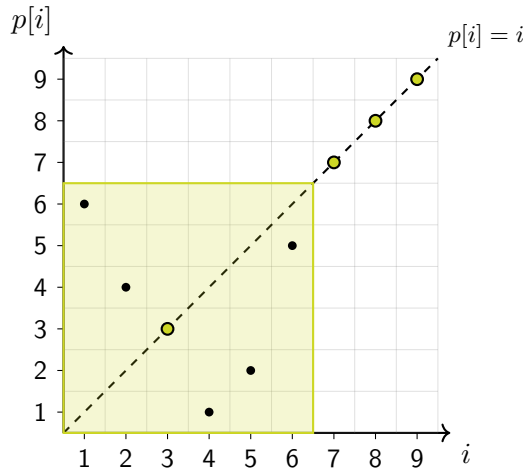
$n \geq 19, n - m \geq 10$.

First, solve $(18, i)$ for $1 \leq i \leq 8$ (for each i , we have ≥ 2000 solutions). Transform these solutions into solutions to (n, m) .

K: Keygen 3



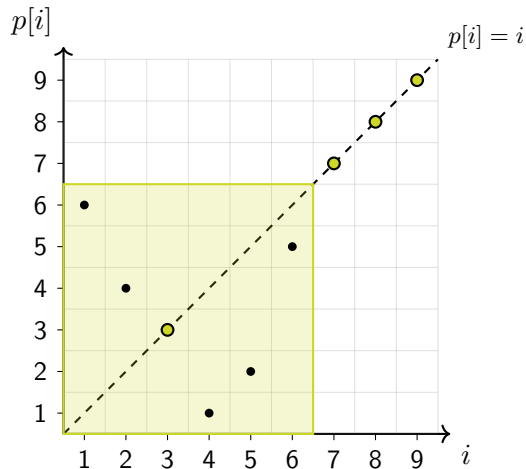
K: Keygen 3



$$n \geq 19, n - m \leq 9.$$

$p_1, p_2, \dots, p_n$ must have at least $n - 9$ cycles, so it must have at least $n - 18$ fixed points (i.e., $p_i = i$).

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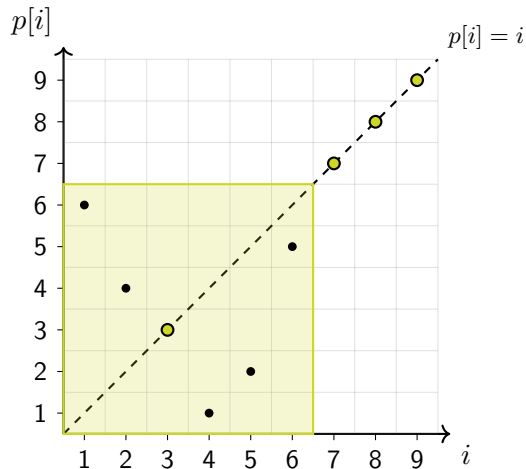
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$[p_1, \dots, p_1 + 1, p_1 + 2, \dots, n]$: the highlighted square has at most 1 fixed point, so there are at most $n - p_1 + 1$ fixed points.

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$[p_1, \dots, p_1 + 1, p_1 + 2, \dots, n]$: the highlighted square has at most 1 fixed point, so there are at most $n - p_1 + 1$ fixed points. Therefore, $p_1 \leq 19$, and we can iterate over all such permutations.